PROBLEMS OF THE CONCENTRATION OF JETS AND THEIR APPLICATION TO THE LABYRINTH SEALS OF TURBOMACHINES

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The potential outflow of an ideal incompressible liquid from a vessel through a slit formed by two flat symmetrical walls with an arbitrary angle between them was investigated in detail by N. E. Zhukovskii, who developed a method for the solution of a similar kind of problem [1].

Let us consider a plane potential jet of an ideal incompressible liquid, flowing toward the slit of two symmetrical branches along the walls of a vessel (Fig. 1a). An analogous problem of a concentrated jet, flowing toward a slit along a bisectorial plane at an angle formed by the walls (Fig. 1b) was discussed in [1], but its solution was not brought down to a numerical result.

We introduce the notation: v, velocity of the liquid at any given point of the jet; V, W, velocity of the liquid at the free boundary of the jet ahead of and behind the slit; $\vartheta = \ln(W/v)$; $\vartheta_0 = \ln(W/V)$; θ, angle between the velocity and the x axis; $2\theta_0$, angle between the walls of the vessel forming the slit; 2b, width of the slit; 2Q, total mass flow rate of the liquid in the jet; φ, potential of the velocities; ψ, stream function.

To seek the equation of the contour of the outflowing jet we use the well-known dependence

$$dy = \frac{1}{W}\sin\theta d\varphi. \tag{1}$$

In accordance with the method of [1], we introduce the complex variable

$$u = r(\cos\lambda + i\sin\lambda) \tag{2}$$

and two auxiliary functions of it

$$\chi(u) = \varphi + i\psi = \frac{Q}{\pi}\ln\frac{u^2 - c^2}{\beta^2} - iQ,$$
$$\Phi(u) = \vartheta + i\theta = iq\int\frac{f du}{\sqrt{(u^2 - f^2)(u^2 - c^2)}}, \tag{3}$$

so chosen that, with real values of u, the variables ψ, ϑ, and θ correspond to the boundaries of the flow under consideration (q, β, and $c < f$ are real numbers) and so that, at the free boundary of the jet COC' inside the vessel $\psi = 0$, and, at its walls and at the external boundaries of the jet outside the vessel $\psi = \pm Q$. In addition, at the free boundaries of the jet under these circumstances the constancy of the velocity is assumed (ϑ = const) and, at the walls, the angle (θ = const). Denoting

$$c = kf, \sqrt{1 - k_2} = k', u = cu', \tag{4}$$

we transform the expression for $\Phi(u)$

$$\Phi(u) = iq\int_0^{u'}\frac{du'}{\sqrt{(1 - u'^2)(1 - k^2u'^2)}} + \omega' q. \tag{5}$$

Here, as in [1], we set ω' equal to the total elliptical integral with the modulus k'.

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Comparing (3) and (5), and bearing in mind that, in the section OC of the contour of the jet, v = V and $\vartheta = \vartheta_0 =$ const, we obtain

$$\vartheta_0 = \ln (W/V) = \omega' q, \text{ i.e., } V = W \exp(-\omega' q). \tag{6}$$

The upper integration limit in expression (5) is the elliptical sine of the value of the integral

$$u' = \mathrm{sn}\left(\frac{\theta}{q} + \frac{\omega' q - \theta}{q} i\right).$$

Taking account of (4) and bearing in mind the relationship $u = r\cos\lambda$ for the boundaries of the flow, it can be shown that, in the section OC, where $0 < r < c$ and (6) is valid,

$$r/c = \mathrm{sn}(\theta/q).$$

The value of θ varies within the limits $0 < \theta < \theta_0$, and θ_0 is found equal to

$$\theta_0 = q\int_0^1 \frac{du'}{\sqrt{(1-u'^2)(1-k^2u'^2)}} = q\omega, \tag{7}$$

where ω is the total elliptical integral with the modulus k.

In the section CF of the wall of the vessel, where $c < r < f$, the angle of the flow remains constant ($\theta = \theta_0 = q\omega$), and the real part of the function $\Phi(u)$ becomes a variable. This corresponds to a change in the velocity v within the limits $V < v < W$, and of the variable ϑ from $\omega' q$ to 0.

In the section Fx of the contour of the jet, after it has issued from the slit v = W, and the angle θ varies within the limits $\theta_0 > \theta > 0$. In this case, $r > f$ and it can be shown, as in [1], that, in this section

$$\frac{f}{r} = \mathrm{sn}\frac{\theta}{q} \quad \text{or} \quad \frac{r}{c} = \frac{1}{k\,\mathrm{sn}\frac{\theta}{q}}. \tag{8}$$

To find the coefficient of the constriction of the jet with its outflow from the slit, we must write an equation for its contour y(x). This can be done, having expression (1) for dy, in which the potential of the velocity φ must be determined. We write an expression for φ, comparing (2), (3), and (8),

$$\varphi = 2\frac{Q}{\pi}\ln\frac{c\,\mathrm{dn}\frac{\theta}{q}}{\beta k\,\mathrm{sn}\frac{\theta}{q}}. \tag{9}$$

Here the symbol dn denotes an elliptical "delta-amplitude" function. Taking account of (9), expression (1) has the form

$$dy = -\frac{2Q}{\pi q}\frac{\mathrm{cn}\frac{\theta}{q}}{\mathrm{dn}\frac{\theta}{q}\cdot\mathrm{sn}\frac{\theta}{q}}\sin\theta\, d\theta,$$

and the equation of the contour of the jet at its outlet from the slit

$$y = -\frac{2Q}{\pi q W}\int_{\theta_0}^{\theta}\frac{\mathrm{cn}\frac{\theta}{q}}{\mathrm{dn}\frac{\theta}{q}\cdot\mathrm{sn}\frac{\theta}{q}}\sin\theta\, d\theta - b.$$

The coefficient of the constriction of the jet μ is the ratio of its width 2y with a sufficiently great distance from the slit ($\theta = 0$) to the width of the slit 2b

$$\mu_1 = \frac{y_{\theta=0}}{-b} = 1 - \frac{2Q}{\pi q W b}\int_0^{\theta_0}\frac{\mathrm{cn}\frac{\theta}{q}}{\mathrm{dn}\frac{\theta}{q}\cdot\mathrm{sn}\frac{\theta}{q}}\sin\theta\, d\theta.$$

At the same time $Q = \mu_1 bW$. Then

$$\mu_1 = 1 - \frac{2\mu_1}{\pi q}\int_0^{\theta_0} \frac{\operatorname{cn}\frac{\theta}{q}}{\operatorname{dn}\frac{\theta}{q}\cdot\operatorname{sn}\frac{\theta}{q}} \sin\theta d\theta,$$

from which

$$\mu_1 = \left(1 + \frac{2}{\pi q}\int_0^{\theta_0} \frac{\operatorname{cn}\frac{\theta}{q}}{\operatorname{dn}\frac{\theta}{q}\cdot\operatorname{sn}\frac{\theta}{q}} \sin\theta d\theta\right)^{-1}. \tag{10}$$

For the problem of Fig. 1b, considered in [1], the coefficient of the constriction of the jet is equal to

$$\mu_2 = \left(1 + \frac{2}{\pi q}\int_0^{\theta_0} \frac{\operatorname{cn}\frac{\theta}{q}\cdot\operatorname{dn}\frac{\theta}{q}}{\operatorname{sn}\frac{\theta}{q}} \sin\theta d\theta\right)^{-1}. \tag{11}$$

We transform expressions (10), (11) to a form convenient for computer computations. To this end, we use the well-known expansions

$$\frac{\operatorname{cn}\frac{\theta}{q}}{\operatorname{dn}\frac{\theta}{q}\cdot\operatorname{sn}\frac{\theta}{q}} = \frac{\pi}{2\omega}\left[\operatorname{ctg}\frac{\pi\theta}{2\omega q} - 4\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi\theta}{\omega q}}{(-1)^n\exp(n\pi\omega'/\omega)+1}\right],$$

$$\frac{\operatorname{cn}\frac{\theta}{q}\cdot\operatorname{dn}\frac{\theta}{q}}{\operatorname{sn}\frac{\theta}{q}} = \frac{\pi}{2\omega}\left[\operatorname{ctg}\frac{\pi\theta}{2\omega q} - 4\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi\theta}{\omega q}}{\exp(n\pi\omega'/\omega)+1}\right].$$

Taking account of (6), (7), and the obvious $(-1)^m = \cos n\pi$, this gives

$$\mu_1 = \left(1 + \frac{1}{\theta_0}\int_0^{\theta_0}\operatorname{ctg}\frac{\pi\theta}{2\theta_0}\sin\theta d\theta + S_1\right)^{-1}; \tag{12}$$

$$\mu_2 = \left(1 + \frac{1}{\theta_0}\int_0^{\theta_0}\operatorname{ctg}\frac{\pi\theta}{2\theta_0}\sin\theta d\theta + S_2\right)^{-1}, \tag{13}$$

where

$$\begin{cases} S_1 = \sum_{n=1}^{\infty}\frac{4\sin\theta_0}{n\pi}\left[\left(\frac{W}{V}\right)^{\frac{n\pi}{\theta_0}} + \cos n\pi\right]^{-1}\left[1-\left(\frac{\theta_0}{n\pi}\right)^2\right]^{-1}, \\ S_2 = \sum_{n=1}^{\infty}\frac{4\sin\theta_0\cos n\pi}{n\pi}\left[\left(\frac{W}{V}\right)^{\frac{n\pi}{\theta_0}} + 1\right]^{-1}\left[1-\left(\frac{\theta_0}{n\pi}\right)^2\right]^{-1}. \end{cases} \tag{14}$$

Before giving the results of the computations, we note that, with $V = 0$, expressions (12), (13), in view of $S_1 = S_2 = 0$, go over into that obtained in [1], especially for the case of outflow from the same vessel without an initial velocity

$$\mu = \left(1 + \frac{1}{\theta_0}\int_0^{\theta_0}\operatorname{ctg}\frac{\pi\theta}{2\theta_0}\sin\theta d\theta\right)^{-1},$$

which, for a vessel with a slit in its flat bottom $(\theta_0 = \pi/2)$, gives

$$\mu = \pi/(\pi + 2) \approx 0.611.$$

Taking into consideration that the result of summation in formula (14) for S_1 is positive, while for S_2 it is negative, we find that in the case under consideration (see Fig. 1a) the coefficient of constriction of the jet is less, and, in the case considered in [1] (See Fig. 1b), greater than with outflow without an initial velocity.

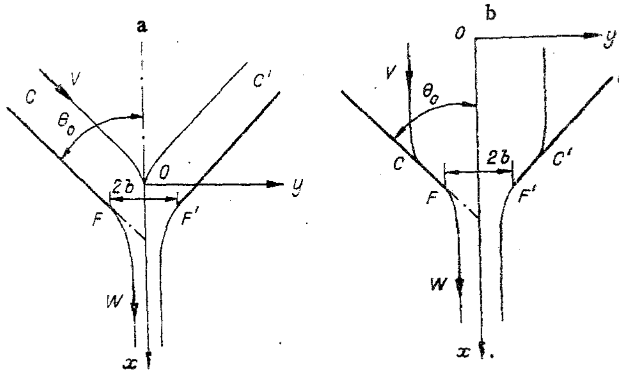

Fig. 1

TABLE 1

Kind of jet	V/W	Values of angle θ_0								
		0	22°30′	45°	67°30′	90°	112°30′	135°	157°30′	180°
At wall (μ_1, Fig. 1a)	0,00	1,000	0,853	0,747	0,669	0,611	0,568	0,538	0,515	0,500
	0,10	1,000	0,853	0,747	0,667	0,605	0,553	0,510	0,475	0,450
	0,20	1,000	0,853	0,746	0,660	0,585	0,520	0,468	0,427	0,400
	0,30	1,000	0,853	0,742	0,643	0,552	0,475	0,417	0,375	0,350
	0,40	1,000	0,853	0,733	0,613	0,505	0,421	0,361	0,322	0,300
	0,50	1,000	0,852	0,712	0,566	0,444	0,358	0,302	0,267	0,250
	0,60	1,000	0,847	0,671	0,497	0,371	0,290	0,241	0,213	0,200
	0,70	1,000	0,831	0,598	0,405	0,287	0,219	0,180	0,159	0,150
	0,80	1,000	0,782	0,476	0,290	0,195	0,146	0,119	0,105	0,100
	0,90	1,000	0,623	0,284	0,154	0,099	0,073	0,059	0,052	0,050
	0,95	1,000	0,428	0,156	0,079	0,050	0,036	0,029	0,026	0,025
	0,99	1,000	0,139	0,034	0,016	0,010	0,007	0,006	0,005	0,005
	1,00	1,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
Bisectorial (μ_2, Fig. 1b)	0,00	1,000	0,853	0,747	0,669	0,611	0,568	0,538	0,515	0,500
	0,10	1,000	0,853	0,747	0,670	0,617	0,584	0,562	0,554	0,550
	0,20	1,000	0,853	0,748	0,677	0,636	0,615	0,602	0,600	0,600
	0,30	1,000	0,853	0,751	0,693	0,666	0,656	0,650	0,650	0,650
	0,40	1,000	0,853	0,760	0,719	0,705	0,702	0,700	0,700	0,700
	0,50	1,000	0,854	0,779	0,756	0,751	0,750	0,750	0,750	0,750
	0,60	1,000	0,859	0,809	0,801	0,800	0,800	0,800	0,800	0,800
	0,70	1,000	0,872	0,851	0,850	0,850	0,850	0,850	0,850	0,850
	0,80	1,000	0,903	0,900	0,900	0,900	0,900	0,900	0,900	0,900
	0,90	1,000	0,950	0,950	0,950	0,950	0,950	0,950	0,950	0,950
	1,00	1,000	1,000	1,000	1,000	1,000	1.000	1,000	1,000	1,000

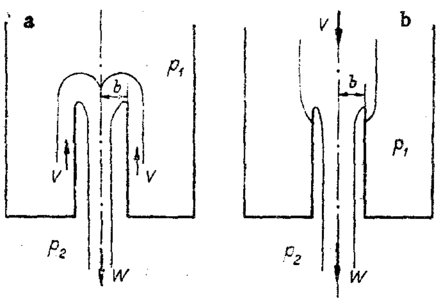

Fig. 2

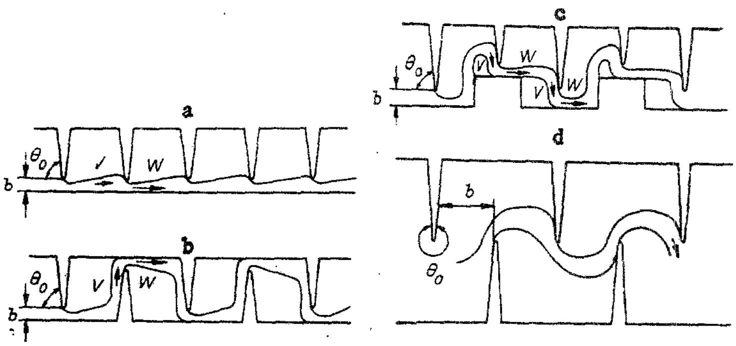

Fig. 3

In the other limiting case where V = W we obtain $\mu_1 = 0$. The value of μ_2 in this case is not immediately evident, and only computations show that $\mu_2 = 1$.

The results of computations using a Nairi-2 digital computer are given in Table 1, where they are shown in the form of a function of the half-angle between the walls θ_0 and the ratio of the initial to the final velocities V/W. The computations showed that an increase in the initial velocity has a different effect on the coefficient of constriction with different schemes of the concentration of the jets. In the case of a jet near the wall, ahead of the slit an increase in its velocity considerably decreases the cross section of the outflowing stream, by the same token limiting its mass flow rate (with $V \to W$, in view of $\mu_1 \to 0$, there is something like self-plugging). In the case of a central jet, ahead of the slit an increase in its velocity increases the cross section of the outflowing stream and promotes an increase in its mass flow rate.

From Table 1 it also follows that, with an increase in the initial velocity along the walls (see Fig. 1a), the effect of the angle between them on the coefficient of constriction of the jet is mainly reinforced, while, with an increase in the veloicty along the bisectorial plane (see Fig. 1b), the effect of the angle between the walls is weakened.

As is well known [2, 3], with outflow through a Borda mouthpiece ($\theta_0 = \pi$) the coefficient of constriction can also be found without using functions of a complex variable. If we use the Bernoulli equation and the equation of the momentum of the liquid for the volume of the jet before and after the mouthpiece, at a sufficient distance from the mouthpiece in both cases illustrated in Fig. 2a, b (in the cases of countercurrent and cocurrent flow, respectively), and, in this case, if it is taken into consideration that the cross section of half the jet following the mouthpiece at a sufficient distance from it is equal to μb and, ahead of the mouthpiece, to $\mu bW/V$, then, we can obtain the values of the coefficients of constriction

$$\mu_1 = (1 - V/W)/2, \ \mu_2 = (1 + V/W)/2.$$

It can be shown that these expressions are valid also if the mass flow rate of the liquid in the oncoming jet is greater than in the jet flowing out through the mouthpiece and, in particular, with an infinite width of the oncoming flow. The latter coincides with the case of outflow into a tube from a continuous stream in a channel of infinite width [4], for which an identical solution has been obtained.

One region of the practical use of the results obtained is the analysis of the flow of a medium in the labyrinth seals of turbomachines.

Figure 3 shows schemes of the flow in different types of seals, involving problems of the concentration of jets. The problem discussed by N. E. Zhukovskii is realized in a direct-flow seal (Fig. 3a). The problem solved by the present author is realized in stepped seals ($\theta_0 \approx 90°$, Fig. 3b, c), as well as in seals of the Keller type, with an overlap close to zero ($\theta_0 \approx 180°$, Fig. 3d). The theoretical results obtained above explain earlier-noted paradoxical experimental data [5, 6], according to which the coefficient of the mass flow rate of a multichamber stepped seal (0.3-0.5) is less than for a single slit (≥ 0.611). In addition, an explanation has been found for the surprisingly high efficiency of a seal of the Keller type near zero overlap [7]. Both of the above-noted phenomena are explained by the fact that, in distinction from single slits, where the outflow, as a rule, takes place without an initial velocity, in multichamber seals, as a result of the incomplete damping of the velocity ahead of the inlet to each succeeding slit, there is flooding of the jet, increasing the through flow in direct-flow seals, and decreasing it in stepped seals, particularly with a rotation by 180° (Keller seal).

The use of calculated values of the coefficients of constriction of jets offers the possibility of refining considerably the calculation of labyrinth seals in the heat-transfer section, of determining the through flow, of finding the conditions for stabilization of the flow, and the distribution of the pressures along the labyrinth and around the periphery of the rotor, carrying the seal.

The adopted model of flow in a seal in the form of a flooded jet with stagnant zones is in good coincidence with experiment on the value of the through flow [8], although visualization shows the presence of vortexes [7, 9], and further refinement can be afforded by the use of models with different types of vorticity in the breakaway zones [10].

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INTERACTION OF HYPERSONIC MULTIPHASE FLOWS

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The investigations of multiphase flows, pursued vigorously in recent years, stem from the practical importance of problems such as supersonic combustion, erosion of materials exposed to flow, various problems of chemical technology, etc.. These flows are also of great interest from the viewpoint of building high-enthalpy gasdynamic facilities [1], which would, in principle, offer modeling of the most important flight parameters of hypersonic vehicles. The basic gasdynamic problem in these areas is to arrange the process of mixing a group of solid (or liquid) particles, accelerated by a light gas, with the supersonic quasiauxiliary flow in which one can excite internal, particularly vibrational, degrees of freedom. The solution of the complete problem can be divided into a number of stages. The first problem is to accelerate solid particles to hypersonic speeds. When the mass ratios of the accelerating and accelerated components are close, the light-gas temperature must be low enough so that the vapors formed in the acceleration (in the case where the particles may vaporize) should not harm the carrier properties of the light gas. It is important to achieve maximum velocities of the solid particles and uniform distribution across the accelerating nozzle. The second task is to examine the mixing process with a view to minimizing perturbation associated with percolation of the particles, their dynamic motion and possible vaporization. Nonuniformities can arise in the flow from several causes: shock waves of various strengths, turbulent fluctuations, etc. To minimize perturbations one must, firstly, so choose the parameters of the interacting gas components and their encounter angle, so that a shock wave does not arise in one of the flows, which may be, e.g., air (Fig. 1). Such a shock wave, however, may be formed for another reason: Because of penetration and vaporization of particles additional perturbations arise, associated with the supply of mass, momentum and energy. Here the macroscopic parameters vary in the mixture. When one cannot achieve conditions for quasiauxiliary flow (i.e., the flow velocity of the gas into which the particles are introduced equal to the tangential component of the particle velocity), additional acceleration of the particles occurs in a certain layer, accompanied by dissipative irreversible processes. There may also be rapid relaxation of vibrational energy in the layer; in addition, the layer may be a source of additional wavelike perturbations.

Thus, both the acceleration of particles, and analysis of the processes occurring inside the zone where the particles mix with the gas stream, are important and independent tasks. We shall examine them in succession. It is known that one can obtain aerosols by using the phenomenon of condensation in a supersonic

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